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A NOVEL METHOD FOR IDENTIFYING VORTICAL STRUCTURES

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A method for identifying vortices in a 2-D velocity field is developed. A function based on the local angles of velocity with respect to a datum is calculated at each point in the 2-D field. This function is a local minimum at vortex centres. An advanced statistical technique for identifying multiple minima is used to find the minima of this function. The method is used for vortices, calculated using a RANS CFD code (CFX 4.3), forming around a very large crude carrier (VLCC) and the results are seen to be close to the centres of the vortices.

1. INTRODUCTION

The identification of both the size and location of vortices are of importance when an engineer wishes to alter the aero- or hydrodynamic characteristics of an object, such as minimizing the drag or changing the vibration behaviour. Where there are a large set of records (either experimental or CFD), there is clearly benefit in an automated system for the identification of vortices. A novel approach which does not require knowledge of the derivative flow quantities is developed here.

The definition of a vortex has been an area of discussion (Miura & Kida 1998) and a number of different vortex definitions have been offered (Jeong & Hussain 1995; Banks & Singer 1995), varying between mathematical and physical descriptions. The specific definition by Lugt (1983) that "*A vortex is the rotating motion of a multitude of material particles around a common centre*" is used in the context of this work. The mathematical definitions that have been proposed usually involve criteria which are functions or components of the velocity gradient tensor, $\nabla \mathbf{u}$. It is common for vortex identification methods to use a combination of two criteria so as to reduce the likelihood of misclassification (Banks & Singer 1995). The majority of vortex identification methods can be split into one of the following groups: Vorticity Maxima (Lesieur *et al.* 2000), Pressure

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Minimum (Banks & Singer 1995), Closed or Spiralling Streamlines (Lugt 1983), Velocity Gradient Tensor (Jeong & Hussain 1995), and Statistical Identification Methods (Kopp *et al.* 2000). Reviews of these techniques can be found in Jeong & Hussain (1995) and Banks & Singer (1995). The VORTFIND method described in the following section can be classed as a Statistical Identification Method.

2. VORTFIND METHOD

We consider a 2-D slice of fluid, approximately normal to the axis of rotation, and the x, y, u and v at a number of points in this plane are processed (where u and v are the velocities in the x and y directions, respectively). The x-axis is taken as a datum and α, which is the angle that the resultant vector of u and v makes to this datum, is calculated at each data point. We then associate with each data point a value β according to the angle α as in Table 1.

At each data point we search for the two nearest points with different values of β to it. The distance to these nearest points are p, q or r for $\beta = 0$, 1 or 2, respectively. For each point, one of p, q or r will always be zero, for its own value of β. Figure 1 is a schematic of this process. Once the minimum values for p, q and r have been found, they are then used to compute

$$l = p^2 + q^2 + r^2. \tag{1}$$

Using Lugt's definition, given earlier, a vortex core is the point which is closest to points of differing values of β, i.e., where l is a minimum. The use of the square of the distance improves the definition of the minimum, as in this way points which are further away from the vortex centre are penalised in comparison to points which are closer.

TABLE 1

Relation between α and β

$0^\circ \leq \alpha < 120^\circ$	$\beta = 0$
$120^\circ \leq \alpha < 240^\circ$	$\beta = 1$
$240^\circ \leq \alpha < 360^\circ$	$\beta = 2$

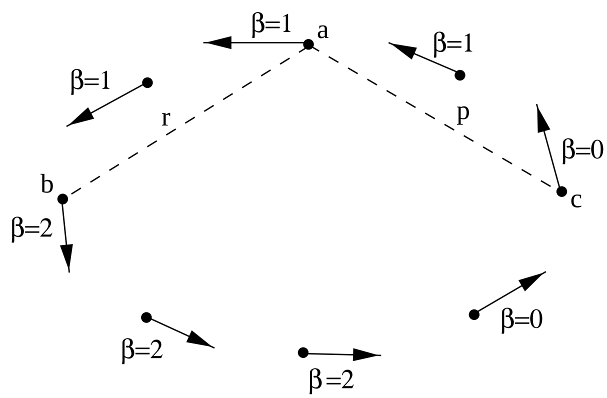

Figure 1. Schematic representation of determination of the value l for an example point a.

In order to predict the locations of the vortices we applied an adaptive Gaussian mixture model (Bishop 1995). The form of such a model is a linear superposition of Gaussian basis functions, i.e.,

$$z(x, y) = \sum_{i=1}^{m} w_i \phi_i(x, y), \tag{2}$$

where $z(x, y)$ is the output of the model at the location x, y in the velocity field, m is the number of components (basis functions), the w_i are a set of weights and the $\phi_i(x, y)$ are adaptive basis functions. We choose the basis functions to be Gaussian functions of the form

$$\phi_i(x, y) = \exp\{-\tfrac{1}{2}([x \quad y]^{\mathrm{T}} - c_i)^{\mathrm{T}} \Sigma_i^{-1}([x \quad y]^{\mathrm{T}} - c_i)\}, \tag{3}$$

where c_i and Σ_i^{-1} are the centre and covariance matrix (which is always nonsingular), respectively, of the ith component, and superscript T denotes the transpose. We assume the covariance matrix is diagonal with

$$\Sigma_i = \begin{bmatrix} \sigma_{i1} & 0 \\ 0 & \sigma_{i2} \end{bmatrix}.$$

The estimation of the model weights, w_i, and the parameters of the components, i.e., $c_i, \sigma_{i1}, \sigma_{i2},\ i = 1, \ldots, m$, is a nonlinear optimization problem for which we applied a simple stochastic search. For a given iteration of the search each parameter was perturbed by a small uniformly distributed random number (uniform in the interval $[-0{\cdot}2, \quad 0{\cdot}2]$). If this leads to a reduction in the mean-squared error after fitting the weights, then the change was accepted; otherwise it was rejected. This procedure was repeated for a predetermined number of iterations. In this way, the Gaussian mixture model adapts itself to the data.

In our case we are seeking to fit the model to the l's. The data were preprocessed as follows.

(i) The 2-D flow was scaled to lie in the unit interval on both axes. This improves the numerical conditioning of the problem and also simplifies the stochastic search as we do not have to take account of different scales in each axis.

(ii) The values of l were scaled by the function

$$l_{\mathrm{new}} = \exp\left\{-\frac{l_{\max}}{2 l_{\min}}\, l\right\},$$

where $l_{\min}$ and $l_{\max}$ are the minimum and maximum values of l, respectively. This had the effect of making vortex cores correspond to maxima and also accentuating the difference between the vortex flows and the surrounding flows. This made it easier for the model to differentiate the vortex locations.

The idea of the algorithm is that, by adapting the mixture of Gaussians, a single Gaussian will eventually be located over each vortex present in the flow. This then provides an indication of the vortex extent and location.

3. RESULTS

The test case chosen for the VORTFIND algorithm was CFD results for the zero Froude number flow around the Kriso Very Large Crude Carrier (VLCC). The flow was solved using the commercial flow solver CFX 4.3 with the standard k–ε turbulence model (CFX 4.3 1998) and the grid was not refined for any particular flow feature. A more detailed description of the calculations can be found in Turnock *et al.* (2000). Figure 3 shows the cross-plane velocity vectors at approximately amidships. One large vortex can be seen to have formed underneath the hull. The second vortex is located on the side of the hull, just above the bilge radius, and is significantly smaller than the first. The 2-D slice considered contained approximately 1 000 data points.

From the CFD data, the values of l were calculated at each grid point. The resulting contour plot of l is shown in Figure 2 together with the estimated locations of the centres

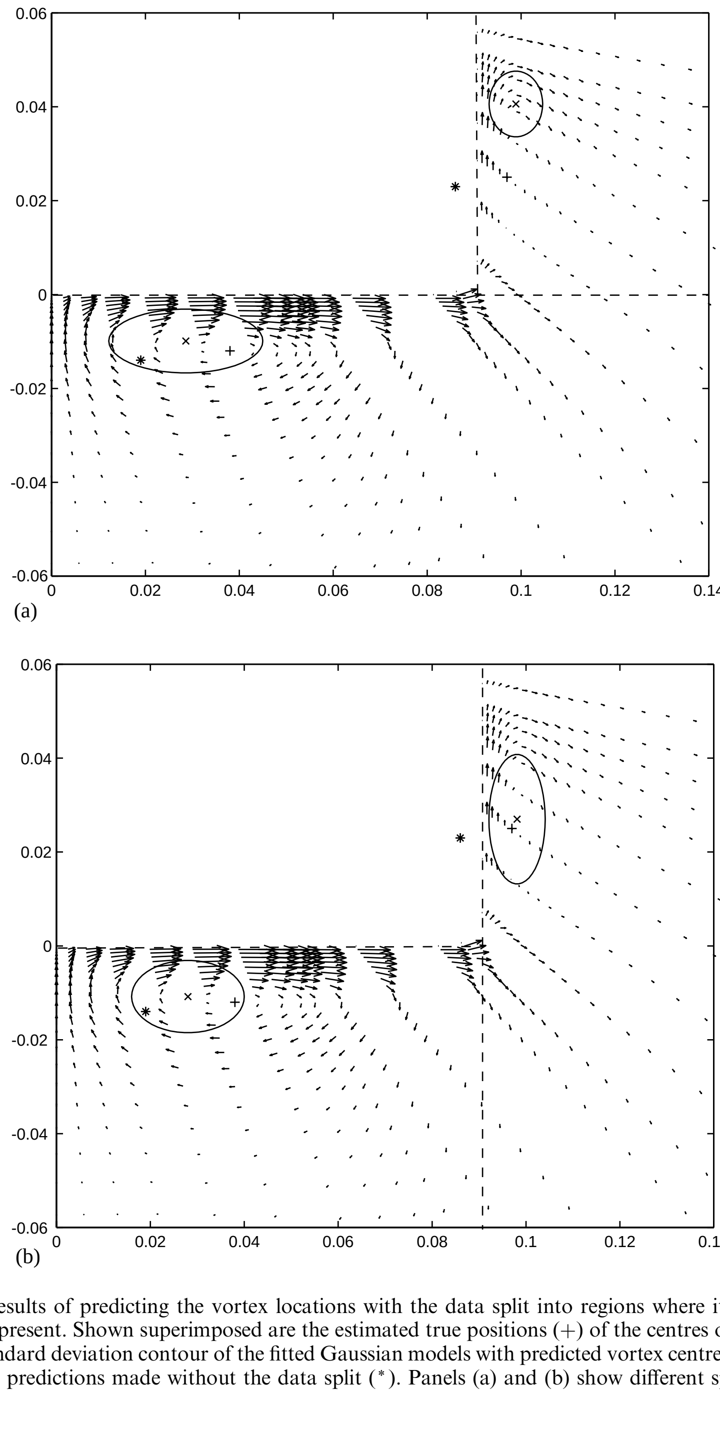

Figure 2. Results of predicting the vortex locations with the data split into regions where it is known only a single vortex is present. Shown superimposed are the estimated true positions (+) of the centres of the two vortices and the one standard deviation contour of the fitted Gaussian models with predicted vortex centres (×). Also shown are the original predictions made without the data split (*). Panels (a) and (b) show different splitting strategies.

of the true vortices and the local minima of l. From this plot it should be evident that l provides only an approximation of where the vortices are. In practise, we cannot define a single point for the vortices due to the grid spacing which prevents very accurate resolution of the vortex centres. Therefore, any method for locating vortices can only be expected to find a region where the vortex is likely to be present.

The adaptive Gaussian model was applied to the data in order to assess the applicability of the approach to locating vortices. Gaussian models with between one and four components were trained as described in Section 2. For each case 10 different random initialisations of the model parameters were used resulting in 10 different models for the vortices. The parameters were adapted over 50 iterations.

The mean squared error fits (over the 10 different runs) were used as criterion to decide when each vortex present in the flow has been identified. At the outset we should expect that the mean squared error (MSE) will always be reduced by the addition of more components as we can then fit the data better (this is up to the point where there are as many components as data points for which the MSE will be zero). What we are therefore looking for is a discontinuity in the gradient of the MSE which will arise at the correct number of components. For the VLCC case, there was a large reduction in the MSE between one and two components and the MSE then decreases at a lower rate. With one component the model is unable to fit the data sufficiently, whereas with the two components a good fit is possible. The models with more than two components can fit the data more accurately but the improvements are less marked. We can therefore reasonably estimate the number of components as two which corresponds to the change in slope of the MSE.

The results of the VORTFIND algorithm are shown in Figure 3 superimposed on the velocity vectors for the actual flows. The centres, as estimated by eye, of the actual vortices are indicated by crosses. The dashed lines indicate where the data was split into two sets, which was done to improve the accuracy of the prediction. When the data is taken as one set, there is a large area with no data points in the region of the ship hull. Although the

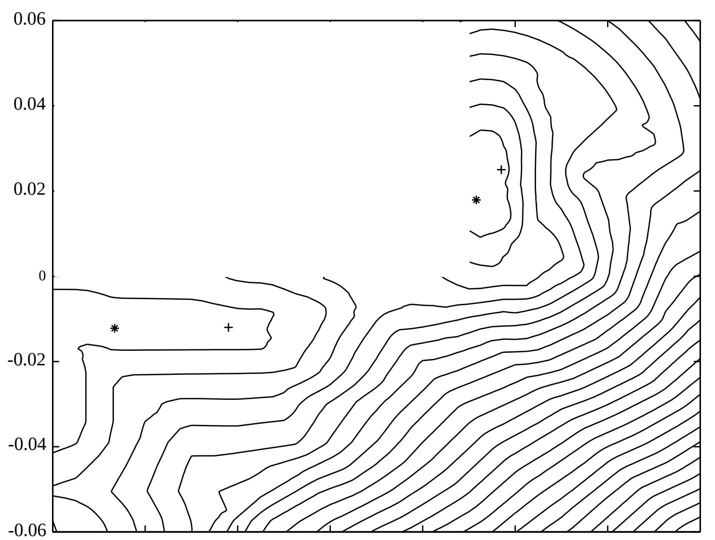

Figure 3. Contour plot of the values of l together with the estimated locations of the centres of the true vortices (+) and the location of the local minimum values of l (*).

location of the vortices were being reasonably well predicted (the points marked with an asterisk), the void of data points was biasing the vortex predictions. One attempt to overcome this problem was to fill the void area with data points whose values were set to the maximum value of l from the true data points. This provided a more realistic boundary condition since there will be no vortex in the hull. There was a slight improvement but splitting the data into two sets was found to give a greater improvement (the points marked with a $\times$). This can partly be attributed to the fact, that when more than one vortex is present, the fitted models are actually trying to model both vortices, and therefore each component will not just be modelling one vortex but a combination of vortices. This may then distort the positions of the components.

Note that in assessing the accuracy of the predictions we must take into account the grid spacing, which makes accurate prediction of the vortex centres by eye difficult and prone to error.

4. CONCLUSIONS

As can be seen in Figure 3 this method identifies the centre of vortices to within a reasonable degree of accuracy for tasks such as grid adaptation for CFD calculations. The accuracy of this method could be improved with further development, for example using another search criterion, such as a local velocity minima, along with minimum l to locate the centre of vortices. The larger of the two vortices identified here is a weak vortex in the presence of a shear flow. This is a case which both vorticity maxima methods and pressure minima methods would have difficulty with. The algorithm does not require a structured set of data points or a knowledge of their connectivity; there is also no need to calculate derivative values. This feature could be particularly beneficial for larger data sets, as not all the data points would be required to identify a vortex. Further tests will be carried out in this area to assess the minimum number of data points required to locate the centre of a vortex. Extending the algorithm to 3-D flows will probably be done in a similar manner to the predictor–corrector technique of (Banks & Singer, 1995), where the vortex cores are identified in a sequence of 2-D planes. Future work will involve testing the VORTFIND algorithm against other vortex identification schemes, such as velocity gradient tensor methods, on more standard test cases than the Kriso VLCC tanker. There will also be a development of more computationally efficient search algorithms. Vortices are flow features for which there are mathematical descriptions, the VORTFIND method shows the scope for statistical identification of flow features which have no practical mathematical description.

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